Pearson Edexcel

# Examiners' Report
# Principal Examiner Feedback

## January 2023

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F3 (WFM03)
Paper 01

## Introduction

This paper proved to be a fair test of student knowledge and understanding. There were some challenging questions for higher ability students but there were many accessible marks available to all students who were suitably proficient in topic areas such as hyperbolic functions, conic sections, matrices, vectors and use of calculus.

## Report on Individual Questions

### Question 1

The opening question was on the differentiation of a product that included an inverse trigonometric function. It was rare to see any student fail to score the first mark for achieving an expression of appropriate form but quite a few did then fail to achieve the second mark, often as a result of forgetting to use the chain rule correctly to multiply by "2" when differentiating arcsin $2x$. Occasionally the derivative of arcsin $2x$ was given as $\frac{1}{1-x^2}$ rather than $\frac{1}{1-(2x)^2}$. A small number made a slip simplifying their expression but this was only penalised for the final mark. There were a few incidences of students giving the derivative of arsinh. Almost all students who used a correct expression for the derivative went on to score this final mark.

### Question 2

This question on a hyperbola saw a great deal of good scoring with full marks being commonly awarded.

In part (a) it was very rare indeed to see a student fail to give a correct equation of the second directrix.

Part (b) was very well-answered with most students able to use a correct eccentricity formula to obtain a quadratic in $a^2$ or $e^2$ which was generally solved correctly. A small number replaced $b^2$ with 25 instead of 5 and occasionally the answer for the value of $a$ was given as ±2 instead of just 2. It was disappointing to see a number of students believe that $\frac{a}{e} = \frac{4}{3} \Rightarrow a = 4, e = 3$.

The vast majority of students proceeded in part (c) to use a correct formula to obtain both foci as coordinates.

### Question 3

Scoring was very high on this question which featured an equation in hyperbolic functions. Most opted to directly replace tanh $x$ and sech $x$ with their exponential definitions although some converted – usually correctly - the given equation to one in sinh and cosh. Only rarely were incorrect exponential forms used although the "2" was occasionally lost during the replacement of sech. Almost all then went on to form a quadratic in $e^x$ and slips were not common, although some fell foul of the error of not completely multiplying through when removing fractions. A small number of students elected to square both sides and use hyperbolic identities – there were a few cases of poor squaring and algebraic slips but many fully correct solutions were seen via this method. Unrejected extra solutions were extremely rare.

**Question 4**
This question on an integral that led to an arsinh function was competently handled by most students. Almost all were able to obtain an arsinh expression in part (a) although it was very common to see the multiplier of $\frac{1}{3}$ missing. Students who are not confident in applying a reverse chain rule when integrating would benefit from checking their answer by differentiating it. Some went straight to a logarithmic form. Those who converted their arsinh into a log expression should take care to use the formula on page 9 of the formula book. Some students think that the two expressions given as the results to the integration of $\frac{1}{\sqrt{a^2+x^2}}$ on page 10 are equivalent but they differ by a constant.

Those who scored in part (a) invariably scored in part (b) with the majority applying the limits –2 and 2 appropriately. A very small number noticed that doubling the expression and using the limits 0 and 2 was slightly more convenient. A few seemed to be a bit confused by the difference of two logs of surds and failed to reach the required form of the answer. However, fully correct solutions were very common and the full 5 marks was the modal mark for this question.

**Question 5**
This question on the eigenvalues and eigenvectors of a matrix saw fairly good scoring on the whole but some students succumbed to algebraic slips.

The method was well known, and those who solved det($\mathbf{A}$–2$\mathbf{I}$) = 0 rather than starting with det($\mathbf{A}$–$\lambda\mathbf{I}$) = 0 were slightly less prone to errors with the determinant. The correct quadratic in $a$ was commonly seen and most ignored the additional negative value. A small number of students formed a system of simultaneous equations via $\mathbf{Ax} = 2\mathbf{x}$ but they weren’t always able to progress. Those who obtained the correct value of $a$ usually proceeded to find the other two eigenvalues although there were a number of cases of poor algebra.

Errors were slightly more common with finding the eigenvectors. The method was again well known but slips were seen in solving the simultaneous equations. The overwhelming majority had no issues normalising their vectors.

**Question 6**
This question on finding a surface area by integration was the first on the paper to prove quite discriminating.

In part (a), correct derivatives were fairly common – the involvement of the constant $a$ in the parametric equations sometimes led to errors where students differentiated parts of the expressions with respect to $a$ instead of $\theta$. Those with the correct derivatives were usually able to simplify their expression into one in cos $\theta$ but the need to convert into half angles proved difficult for some with incorrect identities often attempted.

There were not many fully correct solutions to part (b) although the three method marks were often scored. Most did attempt to apply a correct surface area formula although some did not include or mis-substituted $y$. A small number forgot to include or apply the square root. Those who were able to turn the integrand into one in terms of $\frac{\theta}{2}$ usually picked up the second mark but often did not see how to integrate the expression. Attempts at parts rarely made progress but a small number of students successfully deployed sum/product formulae. Those who were able to integrate often lost the last mark due to slips when substituting the limits.

**Question 7**
This vector question saw good scoring for the most part.

In part (a) the need for a vector product was well known. Slips were seen in its calculation and a small number of students did not choose the correct vectors from the plane equation.

There were plenty of fully correct answers to part (b) although the last mark proved to be quite discriminating. Most correctly found the direction of the line and applied an appropriate formula (usually via a scalar product) to find a relevant angle. The most common error was in giving the correct acute angle, with many making errors such as offering 66 instead of the required 90 – 66 = 24 as their final answer. A simple sketch would be of benefit with questions of this nature. A few attempts were seen that used inappropriate vectors.

The full four marks was the modal score for part (c). A variety of methods were seen including using parallel planes and the formulae for perpendicular distance and projecting on to the normal. Some parallel plane approaches did not combine the two distances correctly. Those using the perpendicular distance formula occasionally made a sign slip with the constant in the plane equation. Those using projection were almost always correct. There were some more long-winded attempts that often attempted to find the point of intersection of the perpendicular line and the plane but these were often successful. There were a few cases of misreading the signs of the components of the vectors and the coordinates of the points.

**Question 8**
This reduction formula task proved to be a generally good source of marks although scoring was poor in part (b).

In part (a), most opted for the sensible split and progress from there was good on the whole. Those who could not score beyond the first mark had usually failed to use the chain rule properly on $\cos^{n-1}x$ when applying parts. The alternative split approach of Way 2 was not a good choice here and usually led to abandoned attempts.

Part (b) saw limited scoring because students often failed to provide enough justification for where the given answer came from. These cases of insufficient working included not sufficiently demonstrating how the given reduction formula led to the result, failing to show the calculation of $I_0$ or not clearly indicating that any $\frac{1}{k}[\cos^{k-1}x\sin x]_0^{\frac{\pi}{2}}$ term was zero. A small number of attempts were seen using proof by induction and these had mixed results. A significant number of students made no response to this question part.

Part (c) was a reasonable source of marks even for those who had made mixed or no progress in the earlier parts. The first mark was widely awarded for using the Pythagorean identity and this led to the correct exact value for many. However, this was a "hence" question and some students embarked on applying the reduction formula again instead of using the result from part (b).

**Question 9**
The final question on an ellipse was probably the most challenging and although many fully correct solutions were seen in part (a), progress in the later parts was much more limited.

The first part was of relatively straightforward demand although there were errors in extracting the correct values of $a$ and $b$ from the given ellipse equation. The value of the eccentricity $e$ was occasionally seen with a

$\pm$ sign and this was penalised here. A small number of students used the hyperbola eccentricity formula that was needed in question 2. It is important that when directrices are asked for that equations are given instead of values of $\frac{a}{e}$.

Part (b) was quite discriminating. The simplest method of using the eccentricity definition of an ellipse was not that widely seen and attempts via this route were often let down by a reliance on the two distances having to be the same. A few attempts just wrote down that the sum of the distances was $2a$ = 2 x 3 = 6 but this was insufficient for a three mark question without proof of this result. Another option was to use a single point but this required a statement of how the sum of the distances to the foci is constant for any position of point $P$. There was some success using a general point in parametric form or in terms of $x$ although many were unable to write their individual expressions as the roots of perfect squares.

Part (c) did see a range of mark profiles but these were generally marks only accessed by the most able students. Many did substitute the equation of the line into the ellipse and simplified the result although some careless algebra was seen. What to do from that point was not well known but those who were able to use half of the sum of the roots to get the midpoint usually obtained the correct locus. Some used $\pm\frac{b}{a}$ or $\frac{b}{2a}$ instead of $-\frac{b}{2a}$ with their quadratic. The question required students to show that the locus was a line passing through the origin and it was unfortunate that many did not make such a conclusion following an otherwise correct solution.